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Intrinsic dissipation in atomic force microscopy cantilevers

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ABSTRACT

In this paper we build a practical modification to the standard Euler–Bernoulli equation for flexural modes of cantilever vibrations most relevant for operation of AFM in high vacuum conditions. This is done by the study of a new internal dissipation term into the Euler–Bernoulli equation. This term remains valid in ultra-high vacuum, and becomes particularly relevant when viscous dissipation with the fluid environment becomes negligible. We derive a compact explicit equation for the quality factor versus pressure for all the flexural modes. This expression is used to compare with corresponding extant high vacuum experiments. We demonstrate that a single internal dissipation parameter and a single viscosity parameter provide enough information to reproduce the first three experimental flexural resonances at all pressures. The new term introduced here has a mesoscopic origin in the relative motion between adjacent layers in the cantilever.

1. Introduction

The Euler–Bernoulli equation is used routinely by the Atomic Force Microscopy (AFM) community to model the flexural dynamics of the cantilever. In this standard approach, dissipation is introduced through terms that address the cantilever–ambient interactions (for example proportional to the velocity works well at high vacuum). Terms of this kind are necessary to account for the finite width of the resonance peaks. However, two problems remain unsolved in this context: one mathematical and one physical. First from the mathematical point of view it is known that a single friction coefficient cannot account for the quality factor (Q) of multiple peaks. If a friction coefficient is chosen to reconstruct the Q of a given peak, then the same coefficient will fail to predict the experimental Q at other resonance frequencies. When needing to work with more than one resonance simultaneously, from a practical point of view one considers multiple friction coefficients. However, this situation suggests a shortcoming of the model, that the dissipation cannot be due to only interaction of the cantilever with the ambient. Second from the physical point of view one knows that even in the absence of external fluid Q remains finite, indicating that other loss mechanisms are at play. Different dissipation mechanisms like the connection of the cantilever with the base and internal processes such as phonon–phonon and phonon–electron scattering as well as thermoelastic damping have been discussed in the literature [1]. In this intrinsic regime below 0.1 mbar (10 Pa), typical Q ranges from the tens [2] to the hundreds of thousands [3]. The task of converting these microscopic mechanisms into useful expressions to be incorporated into the Euler–Bernoulli equation is so daunting that, to our knowledge, it has never been done. Here, recognizing the need to account for internal dissipation, we take an alternative approach based on continuum mechanics through which an appropriate internal dissipation coefficient produces a simple differential term that integrates out the overwhelming information of the microscopic degrees of freedom. Recently we added, on theoretical grounds, an explicit term to the Euler–Bernoulli equation that accounts for internal dissipation in the Atomic Force Microscope cantilever [4]. Our goal in this paper is to firmly establish the validity of the new term based on experimental observations [5]. We have partially tackled this problem, but only for a single instance of a frequency spectrum comprising the lowest two flexural modes [6,7]. Here we use additional extant experimental frequency spectra containing the first three modes as functions of ambient air pressure. We show that a single ambient and a single internal dissipation coefficient are all that is required to produce the correct Q of the three peaks *at all external pressures*.

This work is motivated by the fact that a large number of sample processes studied with AFM are dissipative [8–10], and one commonly is interested in quantifying that dissipation. In those cases, a good understanding of the Q of the cantilever is indispensable in order to subtract its effect from the total Q. The internal dissipation term considered here should be, due to its analytical simplicity, useful for the analysis of experimental data by other groups.

2. Theoretical background

We recall the modified Euler–Bernoulli equation that includes internal dissipation, which we have derived earlier (see Ref. [4]

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for details of the derivation; see also [11] for a discussion of the standard Euler–Bernoulli equation).

$$\rho \frac{\partial^2 u}{\partial t^2} + \frac{EI}{A}\frac{\partial^4 u}{\partial x^4} + \beta \frac{\partial u(x,t)}{\partial t} + \gamma \frac{\partial^4 u}{\partial x^3 \partial t} = 0 \tag{1}$$

Eq. (1) describes the flexion u (x, t) of the AFM cantilever at location x and time t. E represents the elastic modulus, I the area moment of inertia, ρ the density and A the transversal area. The first two terms correspond to inertia and elasticity, respectively. The term β ($\partial u(x, t)/\partial t$) is proportional to the local velocity of the cantilever and represents viscous damping, due to the cantilever–fluid friction. This drag can be written in terms of physically relevant parameters using the Stokes expression [12]

$$\beta = \beta_0 w \eta \rho_A \tag{2}$$

where β_0 is a dimensionless form factor, w is the width of the cantilever, η the ambient viscosity and ρ_A the ambient density. The relevant aspect of Eq. (2) is that the drag coefficient is proportional to the ambient density (or equivalently to the ambient pressure at constant temperature). For the pressures of interest in this work (under 10 Pa) this is consistent with more recent studies [13–15].

The relationship between β and the Q-factor has been studied theoretically, and experimentally [16] in the range $10 \lesssim Q \lesssim 10^4$. In particular, when $\beta \to 0$, $Q \to \infty$. However, under ultra-high vacuum conditions, $\beta \to 0$ as explained above, but Q is known to be finite $(10^3 - 10^5)$ in practice. Therefore a different dissipation term becomes relevant in ultra-high vacuum, when ρ_A and therefore β becomes negligible. It is for this reason that we derived in Ref. [4] the term $\gamma(\partial^4 u/\partial x^3 \partial t)$, which was shown to represent internal dissipation whose physical origin is the relative motion between consecutive layers inside the cantilever.

3. Results

The dispersion relation for a harmonically driven cantilever is obtained by substituting the normal mode solution,$u(x,t) = e^{kx} e^{it\omega}$ into Eq. (1).

$$-\rho\omega^2 + \frac{EI}{A}k^4 + i\omega\beta + i\omega\gamma k^3 = 0 \tag{3}$$

where ω is the angular frequency and k the wavenumber.

Eq. (3) can be readily used to obtain an exact relationship between ω and k. However we exploit the fact that we are interested in the large Q case, to avoid unnecessary complications in the formulas. Thus, the resonance frequencies are

$$\omega_n \approx \sqrt{\frac{EI}{\rho A L^4}} \xi_n^2 \tag{4}$$

where n labels the resonance number, L is the length of the cantilever and we have introduced the notation $\xi = kL$ for the dimensionless wavenumber. For the cantilever clamped at one of its ends and free at the other $\xi_1 = 1.87$, $\xi_2 = 4.69$ and $\xi_3 = 7.85$ for the lowest three resonances [17,18].

From Eqs. 2–4

$$\omega^2 = \frac{EI}{\rho A L^4}\xi_n^4 + i\frac{\sqrt{(EI/\rho A L^4)}\xi_n^2}{\rho}\left(\beta + \frac{\gamma}{L^3}\xi_n^3\right) \tag{5}$$

The quality factor can be computed [19] as the ratio $Re(\omega^2)/Im(\omega^2)$.

$$Q = \frac{\xi_n^2}{\beta_0 w \eta \rho_A L^3 + \gamma \xi_n^3}\sqrt{\frac{L^2 \rho EI}{A}} \tag{6}$$

We are interested in high vacuum, in which case the air density ρ_A can be written in terms of the pressure P and temperature T using the ideal gas law.

$$Q = \frac{\xi_n^2}{(\beta_0 w \eta L^3/\Re T)P + \gamma \xi_n^3}\sqrt{\frac{L^2 \rho EI}{A}} \tag{7}$$

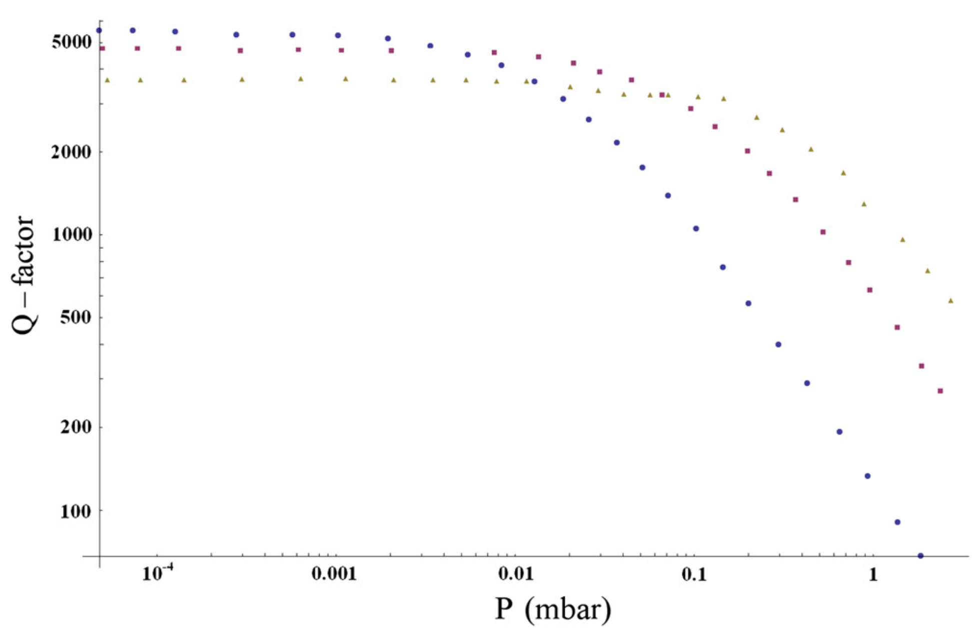

Fig. 1. Quality factor versus pressure for the lowest three resonances of the cantilever. First mode (circles), second mode (squares) and third mode (triangles).

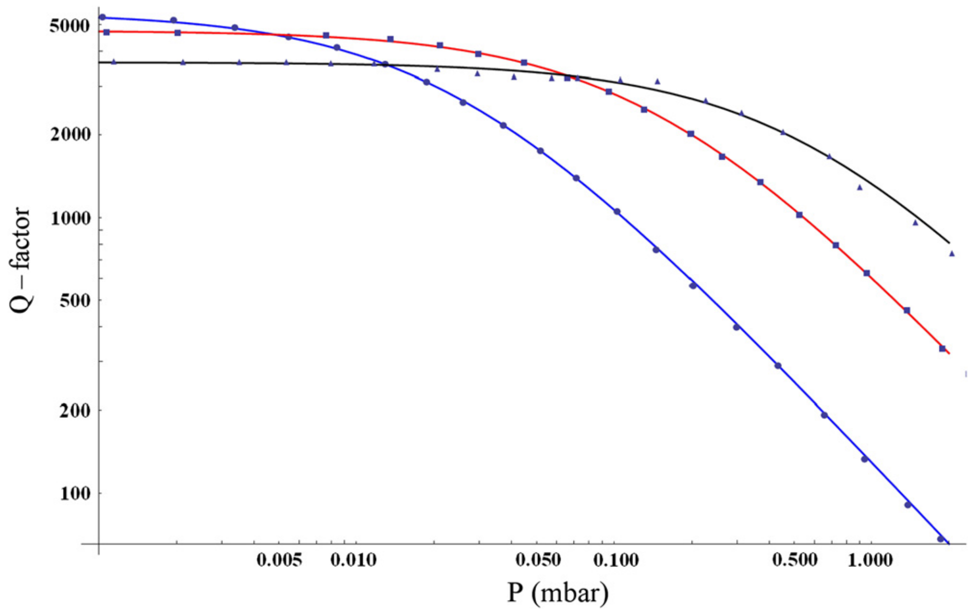

Fig. 2. Comparison of experimental data with theory. Symbols as in Fig. 1. All curves were obtained with the same single viscosity coefficient and the same single internal dissipation coefficient.

where $\Re$ is the gas constant. Eq. (7) can be rearranged thus

$$Q=\frac{\xi_n^2}{[\sqrt{(A/L^2\rho EI)}(\beta_0 w\eta L^3/\Re T)]P+[\gamma\sqrt{(A/L^2\rho EI)}]\xi_n^3} \tag{8}$$

For a given cantilever its intensive and extensive properties are fixed. If in addition the temperature is kept fixed during the experiments then the quantities inside the brackets in Eq. (8) are constants, which we redefine as

$$Q=\frac{\xi_n^2}{BP+\Gamma\xi_n^3}=\frac{1}{(B/\xi_n^2)P+(\Gamma\xi_n)} \tag{9}$$

The parenthesis in the last expression underscores that the denominator is a linear function of the pressure, and the coefficients are constants (B, Γ) simply modified by ξ_n.

This is as much as we can learn of Q from Eq. (1) in the limit of large Q. However, as we mentioned above, for the cantilever fixed at one end there is an additional source of dissipation due to the rotation of the cantilever relative to the base. This dissipation is idiosyncratic, it depends on the connection, and cannot be treated in a general way. To analyze experimental data the term is necessary and we include it in

$$Q=\frac{1}{(BP+\Gamma\xi_n^3)/\xi_n^2)+(1/Q_n^c)} \tag{10}$$

where the dissipation of the connection is represented by Q_n^c, where the subindex n emphasizes that that loss generally depends on frequency.

4. Comparison with experiments

In this section we apply our main result, Eq. (10), to experimental results [5]. An AFM silicon nitride cantilever with dimensions 80 μm length, 17 μm width and 0.2 μm thickness, is forced harmonically at one end while the other end remains free. Fig. 1 shows the data from Ref. [5] for Q versus pressure for the first, second and third resonance peaks.

Fig. 2 shows a comparison between the experiment and the theory developed here. A single set of parameter ($B=(29\pm3)\times10^{-3}$ mbar^{-1}, $\Gamma=(210\pm5)\times10^{-7}$) is enough to correctly reconstruct the *three resonant peaks at all pressures*. Since ultimately Γ must be understood from microscopic considerations, this method can also be seen as a way to measure Γ, useful for eventual checking against microscopic theories.

5. Conclusions

We introduce an internal dissipation in cantilevers, which gives rise to an explicit additional term in the typical Euler–Bernoulli equation of motion. Proper calibration in atomic force microscopy requires an independent Q-factor determination of the excited flexural modes. In particular, in non-contact AFM, where the measurement of the Q due to tip–sample interaction is paramount, the proper subtraction of the intrinsic Q is critical. We tested the validity of the equation by solving it for a free-end cantilever under harmonic excitation and comparing the corresponding theoretical signal with extant experimental curves for silicon nitride cantilevers for a large range of pressures. The agreement is excellent, making the modified Euler–Bernoulli equation (Eq. (10)) an appropriate starting point for AFM studies, particularly in high vacuum, where the new term becomes dominant. We demonstrated that Eq. (10) provides a fit to the three lowest resonant peaks with the same parameters for the three curves. Neither a simple harmonic oscillator model nor ignoring the new term in the Euler–Bernoulli equation can deliver the same results. Thus the new term introduced here has a mesoscopic origin in the relative angular motion between consecutive layers inside the cantilever. The internal dissipation term should be dominant in studies in vacuums with pressures less than about 1 Pa. For pressure higher than that value, external damping due to friction becomes relevant.

Acknowledgments

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